Probabilistic Retrieval

(COSC 488)

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Probabilistic Model

- Use probability to estimate the "odds" of relevance of a query to a document.
  - With having information about relevant and non-relevant sets
  - Without having such information
- Original model (*binary independence model*, *BIM*) does not consider term frequency and document length as parameters of term weight.
- An extended version that includes the document and query term weight has influenced search engines.

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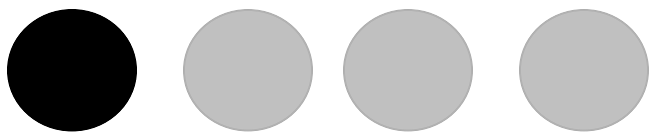
Some Background

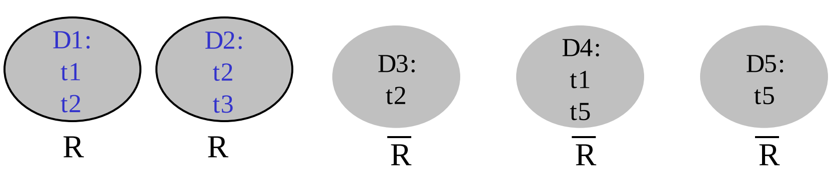
- If we have four balls, three gray and one black, and *it is equally likely that we could pick any of the balls*, we can estimate the probability that of:



- Choosing a black ball = 1/4
- Choosing two black balls in a row (1/4)(1/4) = (1/16)

Relevance Odds for One Term

- We want to estimate, for a given term, the odds of being in a relevant document.



- Assumption: D1 and D2 are relevant; D3, D4 and D5 are non-relevant.
- Need to compute the estimate that a document D_j is relevant given the query term *t1*
- *Odds that* $\mathbf{D_j}$ *is relevant given* t_1 (one solution):

$$O(D_j \mid t1) = \frac{\textit{num relevant with t1 / num relevant}}{\textit{num of docs with t1 / all documents}}$$

$O(D_j \mid t_1) = (1 / 2) / (2 / 5) = 1.25 : 1$

Computing Odds of Relevance for Multiple Terms

- Given query terms $t_1, t_2, ..., t_n$, must compute the odds of relevance given these terms:

$$O(R \mid t_1, t_2, ..., t_n)$$

- Based on the Bayes theorem (independence assumption), we can take the product of these individual odds.

$$O(R \mid Q) = \prod_{i=1}^{i=t} O(R \mid t_i)$$

- Note, since the log function is often used to scale the odds, the sum of the log odds (log of each odds) may be used:

$$\log(\prod_{i=1}^{i=t} O(R \mid t_i)) \approx \sum_{i=1}^{i=t} \log(O(R \mid t_i))$$

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Principles surrounding weights

(Robertson and Sparck Jones, 1976)

- **Independence Assumptions**
  - I1: The distribution of terms in relevant documents is independent and their distribution in all documents is independent.
  - I2: The distribution of terms in relevant documents is independent and their distribution in non-relevant documents is independent.
- **Ordering Principles**
  - O1: Probable relevance is based only on the presence of search terms in the documents.
  - O2: Probable relevance is based on both the presence of search terms in documents and their absence from documents.

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Parameters in Computing Term Weight

N = total number of documents in collection

R = total number of relevant documents for a query

n = number of documents that contain the query term

r = number of relevant documents that contain the query term

Probabilistic Variations to Compute Term Weight

- **I1 and O1:**
  - **Ratio of the ratio of relevant documents having the term to the ratio of all documents having the term**

$$\left(\frac{\frac{r}{R}}{\frac{n}{N}}\right)$$

- **I2 and O1:**
  - **Ratio of the ratio of relevant docs having the term to the ratio of the non-relevant documents having the term**

$$\left(\frac{\frac{r}{R}}{\frac{n-r}{N-R}}\right)$$

Probabilistic Variations to Compute Term Weight

- **I1 and O2:**
  - **Ratio of the odds of a relevant document having the term (i.e., ratio of relevant documents having the term to not having the term) to the odds of all documents having the term (i.e., ratio of all documents having the term to not having the term)**

$$\left(\frac{\frac{r}{R-r}}{\frac{n}{N-n}} \right)$$

- **I2 and O2:**
  - **Ratio of the odds of a relevant document having the term (i.e., ratio of relevant documents having the term to not having the term) to the odds of all non-relevant documents having the term (i.e., ratio of all non-relevant documents having the term to not having the term)**

$$\left(\frac{\frac{r}{R-r}}{\frac{n-r}{(N-n)-(R-r)}} \right)$$

Probabilistic Variations to Compute Term Weight

- To guarantee that the denominator is never zero, adding a minor 0.5 to all numerators and denominators:

$$\left(\frac{\frac{r+0.5}{R-r+0.5}}{\frac{n-r+0.5}{(N-n)-(R-r)+0.5}} \right)$$

← *Robertson/Spark Jones weight*

a priori Relevance Information

- *a priori* Relevance Information not always known
- In on-line systems not possible to have relevant information as training data (r, R)
- Alternative:
  - Relying on user's feedback
  - Without any relevance information

Probabilistic Retrieval Example with *a priori* Relevance Information

- D1: "Cost of paper is up." (*relevant*)
- D2: "Cost of jellybeans is up." (*not relevant*)
- D3: "Salaries of CEO's are up." (*not relevant*)
- D4: "Paper: CEO's labor cost up." (????)

Term	**Relevant**	**Not relevant**	**Evidence**
paper	1	0	for (strong)
CEO	0	1/2	against
labor	0	0	none
cost	1	1/2	for (weak)
up	1	1	none

Probabilistic Retrieval Example (Cont'd)

- *cost* appears in 1 of 1 relevant document
  - odds are (1+.5)/(0+.5) = 3 to 1 that *cost* will appear
- *cost* appears in 1 of 2 non-relevant documents
  - odds are (1+.5)/(1+.5) = 1 to 1 that *cost* will appear
- If *cost* appears in D, then the odds are 3 to 1 that D is relevant.

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Probabilistic Retrieval Example (Cont'd)

- D1: "Cost of paper is up." (*relevant*)
- D2: "Cost of jellybeans is up." (*not relevant*)
- D3: "Salaries of CEO's are up." (*not relevant*)
- D4: "Paper: CEO's labor cost up." (????)

Term	Odds of Relevance
paper	(1.5/0.5)/(0.5/2.5) = 1.5:1
CEO	(0.5/1.5)/(1.5/1.5) = 0.33:1
Labor	(0.5/1.5)/(0.5/2.5) = 1.66:1
cost	(1.5/0.5)/(1.5/1.5) = 3:1
up	(1.5/0.5)/(2.5/0.5) = 0.6:1

$$O(R \mid Q) = \prod_{i=1}^{i=t} O(R \mid t_i)$$ = **1.5:1** **(RSV)**

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Modifications to Basic Probabilistic Model

- Term frequency and document length are not considered in original probabilistic model (BIM – Binary Independence Model).
- Performed worse than vector space model (VSM).

Thus:

- Modification to Probabilistic model – a non-binary model:
  - Incorporating tf-idf (Croft and Harper, 1979)
  - Incorporating document length (Robertson and Walker 1995)

A Common Approach: BM25

$$SC(Q, D_i) = \sum_{j=1}^{t} w \left(\frac{(k_1 + 1)\, tf_{ij}}{tf_{ij} + \underbrace{k_1 \left(1 - b + b \frac{|D|}{avgdl} \right)}_{K}} \right) \left(\frac{(k_2 + 1) qtf_j}{k_2 + qtf_j} \right)$$

$$w = idf = \log\left(\frac{N - n + 0.5}{n + 0.5} \right)$$

← ***IDF*** *is used and normally defined as this!*

k_1, k_2 and b are parameters to be empirically determined.

k_1:1.2 ; k_2:0-1000; b=0.75 (in many cases)