Sparse Power Efficient Topology for Wireless Networks

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Due to the nodes' limited resource in the wireless ad hoc networks, it is important to maintain only a linear number of links using a localized construction method while still preserving the power-efficient route for any pair of nodes.

We first study the power stretch factor of several well-known sparse proximity graphs including relative neighborhood graph, Gabriel graph and Yao graph. Because these graphs do not have constant degrees, we further propose an efficient localized algorithm to construct a sparse spanner that has both a constant-bounded node degree and a constant bounded power stretch factor. We then consider combining several well-known proximity graphs including Gabriel graph and Yao graph to construct power efficient networks. In addition, we propose to construct a new topology by using the Yao and the reverse of Yao structure. We show that the constructed topology is connected if the unit disk graph is connected. Moreover, it has a bounded node degree.

Our experimental results show that the structures proposed in this paper have bounded unicasting power stretch factor and broadcasting power stretch factor in practice.

Key Words: Ad hoc networks, topology, power consumption, optimization.

1. INTRODUCTION

Due to the nodes' limited resource in wireless ad hoc networks, the scalability is crucial for network operations. One effective approach is to maintain only a linear number of links using a localized construction method. However, this sparseness of the constructed network topology should not compromise too much on the power consumptions on both unicast and broadcast/multicast communications. In this paper, we study how to construct a sparse network topology efficiently for a set of static wireless nodes such that every unicast route in the constructed network topology is power efficient. Here a route is *power efficient* for unicasting if its power consumption is no more than a constant factor of the minimum power needed to connect the source and the destination. A network topology is said to be power efficient if given any two nodes, there is a power efficient route to connect them.

We consider a wireless ad hoc network consisting of a set V of n wireless nodes distributed in a two-dimensional plane. Each wireless node has an omnidirectional antenna. A single transmission of a node can be received by all nodes within its vicinity. In the most common power-attenuation model, the power needed to support a link uv is $\|uv\|^\beta$, where $\|uv\|$ is the Euclidean distance between u and v, β is a real constant between 2 and 5 dependent on the wireless transmission environment. By a proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph*

$UDG(V)$ in which there is an edge uv if and only if $\|uv\| \leq 1$. The size of the unit disk graph could be as large as $O(n^2)$. Given a unicasting or multicasting request, the *power efficient routing problem* is to find a route whose power consumption is within a small constant factor of the optimum route.

Recently, Rodoplu and Meng [12] described a distributed protocol to construct a topology, which contains the minimum power path connecting any pair of nodes in UDG. However, their protocol is not time and space efficient. Then, Li and Wan [9] improved their result by giving an efficient localized algorithm to construct a smaller and efficient network topology with a linear number of edges.

A further trade-off can be made between the sparseness of the topology and its power efficiency. The power efficiency of a topology G is measured by its power stretch factor, which is defined as the maximum ratio of the minimum power needed to support the connection of two nodes in G to the minimum necessary in UDG. Recently, Wattenhofer *et al.* [16] tried to address this trade-off. Unfortunately, their algorithm is problematic and their result is erroneous; see [11].

In this paper, we first consider several well-known proximity graphs including relative neighborhood graph, Gabriel graph and Yao graph [6, 7, 17]. These graphs are sparse and can be constructed locally in an efficient way. We show that the power stretch factor of Gabriel graph is always one, and the power stretch factor of Yao graph is at most a real constant, while the power stretch factor of the relative neighborhood graph could be as large as $n-1$. Since these graphs do not have constant bounded node degrees, we further propose another sparse topology, namely the sink structure, which has both a constant bounded node degree and a constant bounded power stretch factor. An efficient localized algorithm is presented for the construction of this topology. This result was reported by us in [10].

We then present some other localized algorithms to construct sparse and power efficient topologies. We show that several combinations of the Yao graph and the Gabriel graph are connected and power-efficient, and have at most $O(n)$ edges while each node has a bounded out-degree. In addition, we show that $YY(V)$ a connected graph if UDG is connected. We conduct experiments to show that these topologies are power efficient in practice.

The rest of the paper is organized as follows. In Section 2, we first give some definitions and review some results related to the network topology control. In Section 3, we study several geometry structures that may be used for network topology control. We then give an algorithm that constructs a sparse and power-efficient spanner with a bounded node degree. In Section 4, we propose several methods to combine some well-known geometry structures to construct power efficient network topologies. Section **??** shows that all structures have a bounded unicasting, broadcasting power stretch factors and bounded node degree in simulation. We conclude our paper in Section 6 by discussing some possible future research directions.

2. PRELIMINARIES

2.1. Geometry Structures

Let $disk(u,v)$ be the disk using edge uv as a diameter. Let $lune(u,v)$ be the intersection of two open disks centered at u and v with radius $\|uv\|$ respectively.

The *constrained relative neighborhood graph* over a (directed) graph G, denoted by $RNG(G)$, is defined as follows. It has an (directed) edge uv iff $uv \in G$ and there is no point $w \in V$ inside $lune(u,v)$ such that $uw \in G$ and $wv \in G$.

The *constrained Gabriel graph* over a (directed) graph G, denoted by $GG(G)$, has an (directed) edge uv iff $uv \in G$ and the open disk using uv as a diameter does not contain any node w from V with both (directed) edges uw and wv are in G.

The *constrained Yao graph* over a (directed) graph G with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG}_k(G)$, is defined as follows. At each node u, any k equally-separated rays originated at u define k equal cones. In each cone, choose the shortest (directed) edge $uv \in G$, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily. If we add the link $\overrightarrow{vu}$ instead of the link $\overrightarrow{uv}$, the graph is denoted by $\overleftarrow{YG}_k(G)$, which is called the *reverse of Yao graph*. Let $YG_k(G)$ be the undirected graph by ignoring the direction of each link in $\overrightarrow{YG}_k(G)$.

These graphs extend the conventional definitions of corresponding ones for the completed Euclidean graph; see [6, 7, 17]. In all of the definitions, when G is a directed graph, all edges in the defined graphs carry their directions also. All these graphs are subgraphs of G. When G is UDG, we use $RNG(V)$, $GG(V)$ and $\overrightarrow{YG}_k(V)$ to denote the corresponding graphs instead. Obviously, for any graph G, $RNG(G)$ is a subgraph of $GG(G)$. It was proved [11, 17] that $RNG(V) \subseteq YG_k(V)$ for $k \geq 6$. However, generally, it is not always true that $RNG(G) \subseteq YG_k(G)$. Bose *et al.* [4] showed that if $UDG(V)$ is connected, it contains the Euclidean minimum spanning tree $EMST(V)$. Then if $UDG(V)$ is a connected graph, $YG(V)$, $GG(V)$ and $RNG(V)$ contain $EMST(V)$ as a subgraph.

These graphs are sparse: $|RNG(V)| < 3n$, $|GG(V)| < 3n$, and $|\overrightarrow{YG}_k(V)| \leq kn$. Here $|G|$ denotes the number of edges of a graph G. The sparseness implies that the average node degree is bounded from above by a real constant. However, the maximum node degree of $RNG(V)$ and $GG(V)$ and the maximum node in-degree of $\overrightarrow{YG}_k(V)$ could be as large as $n-1$ as shown in Figure 1 (a). It places $n-1$ points of V on the unit circle centered at a node $u \in V$. It is not difficult to show that each edge uv_i, $1 \leq i \leq n-1$, is in $RNG(V)$, $GG(V)$ and $YG_k(V)$.

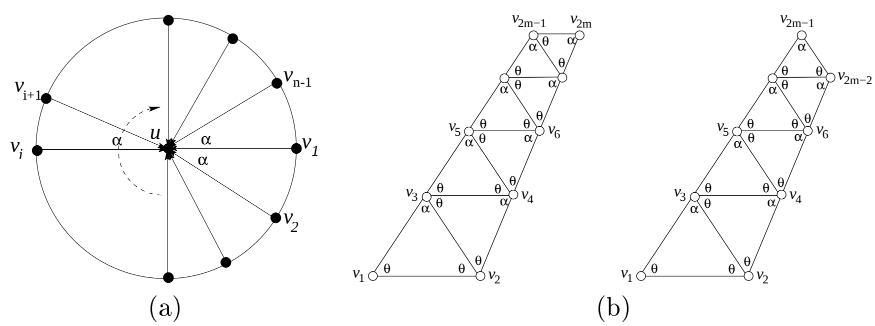


FIG. 1 (a) Node u has degree $n-1$. (b) $RNG(V)$ has large stretch factor.

Figure 1 (a) also shows that no geometry structure with a constant bounded node degree contains the minimum power consumption path for any pair of nodes. If such structure exists, node u in Figure 1 (a) has to maintain each link uv_i, $1 \leq i \leq n-1$, because uv_i is the minimum power consumption path for u and v_i.

The length stretch factor (or dilation ratio, spanning ratio) of a graph G is defined as the maximum ratio of the total edge length of the shortest path connecting any pair of nodes in G to their Euclidean distance. The same analysis by Bose *et al.* [3] implied that the length stretch factor of $RNG(V)$ is at most $n-1$ and the length stretch factor of $GG(V)$ is at most $\frac{4\pi\sqrt{2n-4}}{3}$. It was showed that the Yao graph $YG_k(V)$ has a length stretch factor at most $\frac{1}{1-2\sin\frac{\pi}{k}}$ [8].

Gabriel graph was used as a planar subgraph in [4, 5] to guarantee packet delivery. Relative neighborhood graph was used in [13] for efficient broadcasting in one-to-one broadcasting model.

2.2. Power Stretch Factor

The following definitions are proposed in [10]. However, for the completeness of presentation, we still include them here. Consider any unicast $\Pi(u,v)$ in G (could be directed) from a node $u \in V$ to another node $v \in V$: $\Pi(u,v) = v_0v_1\cdots v_{h-1}v_h,\ where\ u = v_0,\ v = v_h$. Here h is the number of hops of the path Π. The total *transmission power* $p(\Pi)$ consumed by this path Π is defined as $p(\Pi) = \sum_{i=1}^{h} \|v_{i-1}v_i\|^{\beta}$. Let $p_G(u,v)$ be the minimum power consumed by all paths connecting nodes u and v in G. The path in G connecting u, v and consuming the minimum power $p_G(u,v)$ is called the *minimum-power path* in G for u and v. When G is UDG, we will omit the subscript G in $p_G(u,v)$. The *power stretch factor* of graph $H \subseteq G$ with respect to G is defined as $\rho_H(G) = \max_{u,v\in V} \frac{p_H(u,v)}{p_G(u,v)}$. If G is UDG, we use $\rho_H(V)$ instead of $\rho_H(G)$. For integer $n > 0$, let $\rho_H(n) = \sup_{|V|=n} \rho_H(V)$. When the graph H is clear from the context, it is dropped from notation. Li *et al.* [10, 11] proved that for a constant δ, $\rho_H(G) \leq \delta$ iff for any link v_iv_j in graph G but not in H, $p_H(v_i,v_j) \leq \delta\|v_iv_j\|^{\beta}$. It implies that it is sufficient to analyze the power stretch factor of H for each link in G but not in H.

LEMMA 1. *For any $H \subseteq G$ with a length stretch factor δ, $\rho_H(G) \leq \delta^{\beta}$.*

However, the reverse of this lemma is unnecessarily true. Finally, the power stretch factor satisfies monotonic property: If $H_1 \subset H_2 \subset G$ then $\rho_{H_1}(G) \geq \rho_{H_2}(G)$.

2.3. Localized Algorithm

It is preferred that the underlying network topology can be constructed in a localized manner due to the limited resources of the wireless nodes, and possible easy adaption for mobile *ad hoc* network. Stojmenovic *et al.* first defined what is a localized algorithm in several papers [4, 14]. Here a distributed algorithm constructing a graph G is a *localized algorithm* if every node u can exactly decide all edges of G incident on u based only on the information of all nodes within a constant hops of u (plus a constant number of additional nodes' information if necessary). It is easy to see that $YG_k(V)$, $RNG(V)$, and $GG(V)$ can be constructed locally. However, the Euclidean minimum spanning tree $EMST(V)$ can not be constructed by any localized algorithm. In this paper, we are interested in designing localized algorithms to construct sparse and power efficient network topologies.

3. PROXIMITY GRAPHS

In this section, we study the power stretch factor of several sparse geometry structures for UDG. Then we give a new method to construct a sparse network with a bounded node degree and a bounded power stretch factor theoretically.

3.1. Relative Neighborhood Graph and Gabriel Graph

Since the RNG has length stretch factor as large as $n-1$, Lemma 1 implies that its power stretch factor is at most $(n-1)^2$. We show that it is actually $n-1$.

THEOREM 2. $\rho_{RNG}(n) = n-1$.

PROOF. First we prove that $\rho_{RNG}(n)$ is at most $n-1$. Consider the path between u and v in $EMST(V)$. This path contains at most $n-1$ edges and each edge has length at most $\|uv\|$. Therefore its total power consumption is at most $(n-1)\|uv\|^\beta$. Since $EMST(V) \subset RNG(V)$ if $UDG(V)$ is connected, $\rho_{RNG}(n) \leq n-1$.

Then we show that $\rho_{RNG}(n) \geq n-1-\varepsilon$ for any small positive ε by constructing an example illustrated in Figure 1 (b). We consider two cases. We first consider even n, say $n = 2m$. The construction of the point set V is shown in the left figure of Figure 1 (b), which was used in [3]. Let $\alpha = \frac{\pi}{3}+2\delta, \theta = \frac{\pi}{3}-\delta$, where δ is a sufficiently small positive number. The m points with odd subscripts $v_1, v_3, v_5, \cdots, v_{2m-1}$ are collinear, so are the m points with even subscripts $v_2, v_4, v_6, \cdots, v_{2m}$. As proved in [3], $RNG(V)$ is a path $v_1, v_3, v_5, \cdots, v_{2m-1}, v_{2m}, \cdots, v_6, v_4, v_2$. As $\delta \longrightarrow 0$, the length of each edge in $RNG(V)$ tends to $\|v_1v_2\|$ from below, which implies that $\frac{p_{RNG}(u,v)}{p(u,v)} \longrightarrow n-1$, So we can find a sufficiently small $\delta > 0$ such that $\frac{p_{RNG}(u,v)}{p(u,v)} > n-1-\epsilon$, which implies that $\rho_{RNG}(n) > n-1-\epsilon$. When n is odd, the construction is shown in the right figure of Figure 1 (b) and the existence can be proved by a similar argument. ◻

It implies that $\rho_G(n) \leq n-1$ if $EMST \subseteq G$. The Gabriel graph has length stretch factor in between $\frac{\sqrt{n}}{2}$ and $\frac{4\pi\sqrt{2n-4}}{3}$[3]. Obviously, $\rho_{GG}(n)$ is at most $min(\left(\frac{4\pi\sqrt{2n-4}}{3}\right)^2, \rho_{RNG})$.

THEOREM 3. *$\rho_H(GG(H)) = 1$ for any graph H.*

PROOF. Consider any link uv in any minimum power consumption path in H. Then $disk(u,v)$ is empty of wireless nodes w such that uw and wv are both in H. It implies that edge $uv \in GG(H)$. So $\rho_H(GG(H)) = 1$ for any graph H. ◻

3.2. Yao Graph

The Yao graph $YG_k(V)$ has length stretch factor $\frac{1}{1-2\sin\frac{\pi}{k}}$. Lemma 1 implies that its power stretch factor is at most $(\frac{1}{1-2\sin\frac{\pi}{k}})^\beta$. We will prove a stronger result.

THEOREM 4. *The power stretch factor of $YG_k(V)$ is at most $\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$.*

PROOF. It is sufficient to show that for any nodes u and v with $\|uv\| \leq 1$, there is a path connecting them in $YG_k(V)$ with power consumption at most $\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}\|uv\|^\beta$. Let $\delta = \frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$. We construct a path $u \leadsto v$ connecting u and v in $YG_k(V)$ as follows. If link $uv \in YG_k(V)$, then set $u \leadsto v$ as the link uv. Otherwise, there must exist another node w in the same cone as v, which is a neighbor of u in $YG_k(V)$. Then $u \leadsto v$ is set as the concatenation of the link uw and path $w \leadsto v$. Since the angle θ of each cone section is $\frac{2\pi}{k}$, it is easy to show that $\|wv\| < \|uv\|$ when $k > 6$. Consequently, the path $u \leadsto v$ is a simple path, i.e., each node appears at most once. We prove by induction that the path $u \rightsquigarrow v$ has total power consumption $p(u \rightsquigarrow v)$ at most $\delta\|uv\|^\beta$.

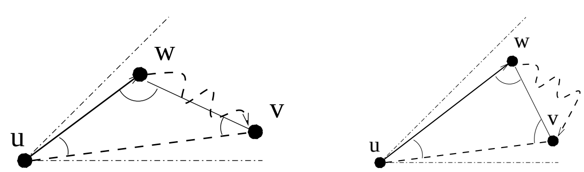
Obviously, if there is only one edge in $u \rightsquigarrow v$, $p(u \rightsquigarrow v) = \|uv\|^\beta < \delta\|uv\|^\beta$. Assume that the claim is true for any path with l edges. Then consider a path $u \rightsquigarrow v$ with $l+1$ edges, which is the concatenation of edge uw and a path $w \rightsquigarrow v$ with l edges. We consider two cases.

Case 1: the angle $\angle uwv$ is not acute. We have $\|uw\|^2 + \|wv\|^2 \leq \|uv\|^2$. Notice that $\frac{\|uw\|}{\|uv\|} \leq 1$ and $\frac{\|wv\|}{\|uv\|} \leq 1$. It implies that,

$$\left(\frac{\|uw\|}{\|uv\|}\right)^\beta + \left(\frac{\|wv\|}{\|uv\|}\right)^\beta \leq \left(\frac{\|uw\|}{\|uv\|}\right)^2 + \left(\frac{\|wv\|}{\|uv\|}\right)^2 \leq 1$$

Therefore, $\|uw\|^\beta + \|wv\|^\beta \leq \|uv\|^\beta$ for any $\beta \geq 2$. Notice that $\|wv\| < \|uv\| \leq 1$, which implies that we can apply induction on the path $w \rightsquigarrow v$ also. See Figure 2 (a). Therefore, $p(w \rightsquigarrow v) \leq \delta\|wv\|^\beta$ by induction. Then

$$p(u \rightsquigarrow v) = \|uw\|^\beta + p(w \rightsquigarrow v) \leq \|uw\|^\beta + \delta\|wv\|^\beta \leq \delta\|uv\|^\beta.$$



(a) $\angle uwv \geq \frac{\pi}{2}$ (b) $\angle uwv < \frac{\pi}{2}$

FIG. 2 Configurations of u, v, w.

Case 2: the angle $\angle uwv$ is acute. We bound the length $\|wv\|$ respecting to $\|uv\|$. See Figure 2 (b). The maximum length of vw is achieved when $\|uw| = \|uv\|$ because the angle $\angle uwv$ is acute. Thus, $\|wv\| \leq 2\sin\frac{\theta}{2}\|uv\| = 2\sin\frac{\pi}{k}\|uv\|$. By induction,

$$\begin{aligned} p(u \rightsquigarrow v) &= |uw|^\beta + p(w \rightsquigarrow v) \leq \|uw\|^\beta + \delta\|wv\|^\beta \\ &\leq \|uv\|^\beta + \frac{1}{1-(2\sin\frac{\pi}{k})^\beta}(2\sin\frac{\pi}{k})^\beta\|uv\|^\beta = \frac{1}{1-(2\sin\frac{\pi}{k})^\beta}\|uv\|^\beta. \end{aligned}$$

This finishes the proof. ◻

The Yao graph has some other nice properties which are important for constructing wireless network topology besides a bounded power stretch factor. Each node has a bounded out-degree. It can be constructed efficiently even when the wireless nodes are not static. For mobile wireless network, there are three events that will possibly trigger the change of the Yao structure, namely, a node leaving the transmission range, a node entering the transmission range, and a node switching the cone region. Updating the Yao structure is fast in all these three scenarios.

3.3. Bounded Degree Spanner

Notice that although the directed graph $\overrightarrow{YG}_k(V)$ has a bounded stretch ratio and a bounded out-degree k for each node, some nodes may have very large in-degrees. The node configuration given in Figure 1 will result a very large in-degree for node u. Bounded out-degree gives us advantages when apply several routing algorithms. However, unbounded in-degree at node u will often cause large overhead at u. Therefore it is often imperative to construct a sparse network topology such that both the in-degree and the out-degree are bounded by a constant while the topology is still power-efficient.

Arya *et al.* [1] gave an ingenious technique to generate a bounded degree spanner. We apply the same technique to construct a sparse network topology with a bounded node degree and a bounded power stretch factor. The technique is to replace the directed star consisting of all links toward a node u by a directed tree $T(u)$ of a bounded degree with u as the sink. Tree $T(u)$ is constructed recursively.

First, we compute the graph $\overrightarrow{YG}_k(V)$. Each node u will have a set of incoming nodes $I(u) = \{v \mid \overrightarrow{vu} \in \overrightarrow{YG}_k(V)\}$. Choose the same k equal-sized cones centered at u: $C_1(u)$, $C_2(u)$, $\cdots, C_k(u)$. In each of the k cones, node u finds the nearest node $y_i \in I(u)$, $1 \leq i \leq k$, if there is any. Link $\overrightarrow{y_i u}$ is added to $T(u)$ and y_i is removed from $I(u)$. For any newly added v in the order of their appearances in $T(u)$, let w be its parent in $T(u)$ and choose the nearest node of $I(u) \cap C_i(w) - \{v\}$ in each cone $C_i(v)$, $1 \leq i \leq k$, centered at v. Then create directed links from the found nodes to v and remove the found nodes from $I(u)$. The process is terminated when $I(u)$ becomes empty. Here u constructs the tree $T(u)$ and then broadcasts the structure of $T(u)$ to all nodes in $T(u)$. Figure 3 (a) illustrates a directed star centered at u and Figure 3 (b) shows the directed tree $T(u)$ constructed to replace the star. The union of all trees $T(u)$ is called the *sink structure*, denoted by $\overrightarrow{YG}^*_k(V)$.

(a) The directed star toward u. (b) The directed tree $T(u)$.

FIG. 3 The directed star and tree for a node u.

THEOREM 5. *The power stretch factor of $\overrightarrow{YG}^*_k(V)$ is at most $\left(\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}\right)^2$. The maximum node degree of $\overrightarrow{YG}^*_k(V)$ is at most $(k+1)^2 - 1$.*

PROOF. Using the same argument as Theorem 4, we can prove that, for each node $v \in I(u)$, there is a direct path $\Pi_{T(u)}(v, u)$ in $T(u)$ such that the power consumption of $\Pi_{T(u)}(v, u)$ is no more than $\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}\|vu\|^\beta$. It implies that the power stretch factor of the graph $\overrightarrow{YG}^*_k(V)$ is at most $\left(\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}\right)^2$.

Notice that the sink structure does not change the out-degree of a node. One directed edge $\overrightarrow{vu}$ implies that there is also one directed edge $\overrightarrow{vw}$ in $T(u)$ for some $w \in I(u)$. Moreover, each node v participates in at most $k+1$ sink trees. It will participate at most k sink trees for some other nodes and itself will also have one sink tree $T(v)$. Node v participates in one sink tree will introduce at most k in-degrees, the total in-degree is therefore at most $k(k+1)$. Consequently, the total degree is at most $k(k+1) + k = (k+1)^2 - 1$. $\square$

Notice that the sink structure and the Yao graph structure do not have to have the same number of cones. For setting up a power-efficient wireless networking, each node u finds all its neighbors in $YG_k(V)$, which can be done in linear time proportional to the number of nodes within its transmission range. The other efficient way is to send a *hello* message to nodes within each cone region using power incrementally until one closest node in that cone is found, if there is any.

4. COMBINING PROXIMITY GRAPHS

In this section, we propose several methods that combine some of these proximity graphs together, to generate practically good network topologies in practice.

4.1. Yao and Gabriel Graph

We give two methods to combine the Gabriel and the Yao structures. First, to further reduce the number of edges, we can apply the Gabriel graph structure to the constructed Yao graph $\overrightarrow{YG}_k(V)$. A directed edge $\overrightarrow{uv}$ in $\overrightarrow{YG}_k(V)$ survives if $disk(u,v)$ does not contain w with $\overrightarrow{uw}$ and $\overrightarrow{wv}$ in $\overrightarrow{YG}_k(V)$. The power stretch factor of the constructed network topology is also at most $\frac{1}{1-(2\sin\frac{\pi}{k})^{\beta}}$ and the out-degree of each node is at most k. Let $\overrightarrow{GYG}_k(V) = GG(\overrightarrow{YG}_k(V))$ be the constructed topology. The number of edges of $\overrightarrow{GYG}_k(V)$ is bounded by $O(kn)$.

On the other hand, we can apply the Yao structure over the Gabriel graph, denoted by $\overrightarrow{YGG}_k(V) = \overrightarrow{YG}_k(GG(V))$. Because $GG(V)$ has a power stretch factor equal to one, the power stretch factor of $\overrightarrow{YGG}_k(V)$ is therefore the same as that of $\overrightarrow{YG}_k(V)$. The node out-degree of $\overrightarrow{YGG}_k(V)$ is bounded by k. Moreover, the number of edges in $\overrightarrow{YGG}_k(V)$ is bounded by $3n$, which is a bound on the number of edges in $GG(V)$.

It is easy to show that graphs $\overrightarrow{GYG}_k(V)$ and $\overrightarrow{YGG}_k(V)$ are both connected if $UDG(V)$ is connected and $k > 6$. The proof is omitted here due to space limit.

The performances of these heuristics will be studied in Section 5. Wattenhofer *et al.* proposed a similar two-phased approach, consisting of a variation of the Yao graph followed by a variation of the Gabriel graph. They tried to prove that the constructed spanner has a constant power stretch factor and the node degree is bounded by a constant. Unfortunately, there are some bugs in their proof and their result is erroneous, which were discussed in detail in [10].

4.2. Yao plus Reverse Yao Graph

In this section, we present a new algorithm that constructs a sparse and power efficient topology. Assume that each node v_i of V has a unique identification number $ID(v_i) = i$. The identity of a directed link $\overrightarrow{uv}$ is defined as $ID(\overrightarrow{uv}) = (||uv||, ID(u), ID(v))$. Then we order all directed links (at most $n(n-1)$ such links) in an increasing order of their identities. Here the identities of two links are ordered based on the following rule: $ID(\overrightarrow{uv}) > ID(\overrightarrow{pq})$ if (1) $||uv|| > ||pq||$ or (2) $||uv|| = ||pq||$ and $ID(u) > ID(p)$ or (3) $||uv|| = ||pq||$, $u = p$ and $ID(v) > ID(q)$. Correspondingly, the rank of each directed link $\overrightarrow{uv}$, denoted by $rank(\overrightarrow{uv})$, is its order in the sorted directed links. Here, we only have to consider the links with length no more than one.

Algorithm 1. *Yao+ReverseYao Topology Construction*

1. Constructs $\overrightarrow{YG}_k(V)$: ties broken by taking the link with the smallest ID.
2. Each node v chooses a node u from each cone, if there is any, so the directed link $\overrightarrow{uv}$ has the smallest $ID(\overrightarrow{uv})$ among all directed links in $\overrightarrow{YG}_k(V)$.

The union of all chosen directed links is the final topology, denoted by $\overrightarrow{YY}_k(V)$. If the directions of all links are ignored, the graph is denoted as $YY_k(V)$. The following theorem is proved in [11]. The proof is omitted here due to space limit.

Theorem 6. *$\overrightarrow{YY}_k(V)$ is strongly connected if UDG is connected and $k > 6$.*

It is obvious that both the out-degree and in-degree of a node in $\overrightarrow{YY}_k(V)$ are bounded by k. And our experimental results show that this sparse topology has a small power stretch factor in practice (see Section 5). We conjecture that it also has a constant bounded power stretch factor theoretically. The proof of this conjecture or the construction of a counter-example remains a future work.

5. EXPERIMENTAL RESULTS

In this section we measure the performances of the new sparse and power efficient topologies by conducting some experiments. An important requirement of all topologies is strong connectivity. We have shown that all these sparse topologies are strongly connected if UDG is connected. So in our experiments, we randomly generate a set V of n wireless nodes and its $UDG(V)$, and test the connectivity of $UDG(V)$. If it is strongly connected, we construct different topologies from V by various algorithms. Then we measure the sparseness and the power efficiency of these topologies by the following criteria: the average and the maximum node degree, denoted by d_{avg}, d_{max}; and the average and the maximum power stretch factor, denoted by ρ_{avg}, ρ_{max}. For a directed topology, we also measure the average and the maximum in-degree, denoted by I_{avg}, I_{max}. In the experimental results presented here, we choose total $n = 100$ wireless nodes in a 10×10 square; set the transmission range to 3; choose 8 cones when we construct any graph using the Yao structure; set the power attenuation constant $\beta = 2$. We generate 1000 vertex sets V (each with 100 vertices) and then construct the graphs for each of these 1000 vertex sets. Figure 4 gives all eight different topologies defined in this paper.

Node Degree. The average node degree of the wireless networks should not be too large. Otherwise a node with a large degree has to communicate with many nodes directly. This increases the interference and collision, and increases the overhead at this node. The average node degree should also not be too small either: a small node degree usually implies that the network has a lower fault tolerance and it also tends to increase the overall network power consumption as longer paths may have to be taken. Thus, the average node degree is an important performance metric for the wireless network topology. Table 1 shows that $GYG(V)$, $YGG(V)$, and $YY(V)$ have a much less number of edges than the $YG(V)$, which is also verified by Figure 4. The in-degree of $YG^*(V)$ is bounded from above by a constant theoretically. Here, O_{max} is the maximum node out-degree over all nodes and all directed graphs.

Power Stretch Factor. Besides strong connectivity, the most important design metric of wireless networks is perhaps the power efficiency, as it directly affects

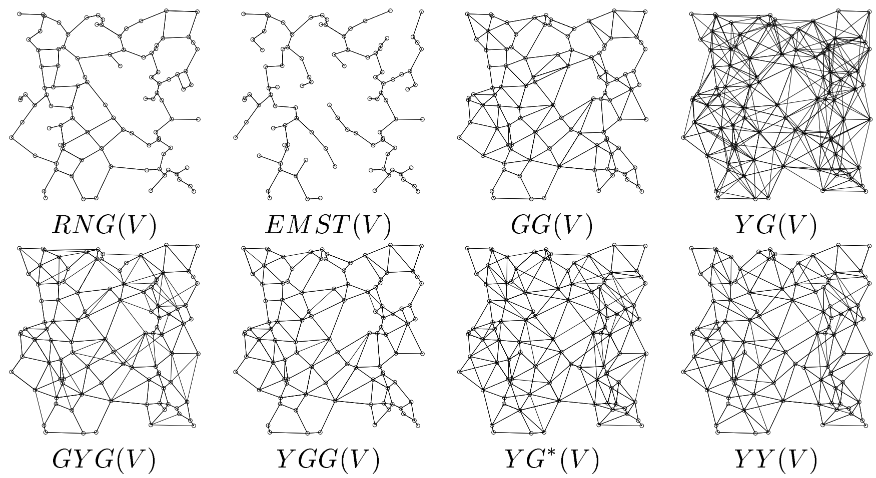


FIG. 4 Different topologies generated from the same $UDG(V)$.

both the node and the network lifetime. So while our new topologies increase the sparseness, how do they affect the power efficiency of the constructed network? Table 1 summarizes our experimental results on the power stretch factors of these topologies. It shows that the new proposed topologies still have small power stretch factors. Notice that although the average and the maximum node degree of $GYG(V)$, $YGG(V)$, and $YY(V)$ are much smaller than that of $YG(V)$, the average and the maximum power stretch factors of these graphs are at the same level of that of $YG(V)$. Especially, the power stretch factor of $YY(V)$ is just a little bit higher than those of $GYG(V)$ and $YGG(V)$. Remember that $YY(V)$ has a bounded node degree while no other topology (except $YG^*(V)$) has such a property.

Broadcasting Power Stretch Factor. The power stretch factor (see Subsection 2.2) discussed previously is defined for the unicast communications. However, in practice, we also have to consider the broadcast or multicast communications. Here, we assume a one-to-all-neighbors communication model. Wan *et al.* [15] proved that the minimum power cost of any broadcasting scheme is at least $\frac{1}{12}\sum_{e\in EMST}||e||^\beta$. Thus, given a topology G, $\sum_{e\in G}||e||^\beta$ could be a good approximation of its performance for broadcasting. The broadcasting power stretch factor, denoted by σ_G, of a topology G over a point set V, is defined as $\sigma_G = \frac{\sum_{e\in G}||e||^\beta}{\sum_{e\in EMST}||e||^\beta}$.

Unfortunately, the broadcasting (or multicasting) power stretch factor of any graph structures mentioned above (except $EMST$) could be $O(n)$. Figure 1 (a) gives such an example of wireless nodes with $\sigma_{RNG(V)} \to n$. On the other hand, our experiments (see Table 1) show that the broadcasting power stretch factors of $GYG(V)$, $YGG(V)$ and $YY(V)$ are actually small enough for practical usage. In Table 1, σ_{avg} and σ_{max} are the average and the maximum multicasting/broadcasting power stretch factor over all graphs respectively.

Notice that Arya *et al.* [1, 2] gave centralized algorithms to construct a graph that has bounded node degree, whose total edge length is no more than a constant factor of that of $EMST(V)$; and it has has a bounded length stretch factor. However, it is complicated to transform their algorithms to distributed ones. It remains open to construct a broadcasting power efficient topology in a localized manner.

TABLE 1
Quality measurements of different topologies.

	d_{avg}	d_{max}	I_{avg}	I_{max}	O_{max}	ρ_{avg}	ρ_{max}	σ_{avg}	σ_{max}
UDG	23.46	50	-	-	-	1.000	1.000	96.756	110.434
GG	3.56	9	-	-	-	1.000	1.000	3.819	4.770
RNG	2.37	5	-	-	-	1.059	3.131	1.694	2.083
$EMST$	1.98	4	-	-	-	1.487	20.788	1.000	1.000
YG	9.05	22	6.66	21	8	1.002	1.555	12.967	15.615
GYG	4.47	12	3.88	10	8	1.002	1.555	5.327	7.118
YGG	3.56	9	3.46	9	8	1.002	1.555	3.628	4.292
YG^*	5.94	12	5.53	13	8	1.003	1.833	7.302	8.937
YY	5.00	11	4.64	8	8	1.004	1.833	6.899	8.492

TABLE 2
Summary of some proximity graphs.

	Length	Power	Degree
RNG	$n-1$	$n-1$	$n-1$
GG	$\frac{4\pi\sqrt{2n-4}}{3}$	1	$n-1$
YG	$\frac{1}{1-2\sin\frac{\pi}{k}}$	$\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$	$n-1$
YG*	$\left(\frac{1}{1-2\sin\frac{\pi}{k}}\right)^2$	$\left(\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}\right)^2$	$(k+1)^2-1$
GYG	$O(\sqrt{n})$	$\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$	$n-1$
YGG	$O(\sqrt{n})$	$\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$	$n-1$
YY	*open*	*open*	$2k$

6. SUMMARY AND FUTURE WORK

In this paper, we consider how to maintain a simple network topologies that are power-efficient. We summarize the results in Table 2.

We proposed several methods to combine some well-known geometry structures such as $GG(V)$ and $YG(V)$ to get the new sparse topologies $\overrightarrow{GYG}(V)$ and $\overrightarrow{YGG}(V)$. They are power-efficient and have constant bounded node out-degrees. We then presented an algorithm to construct a new topology, $YY(V)$, which has a bounded node degree. Our experimental results showed that its power stretch factor is very small in practice. In addition, the experiments showed that these three topologies have small broadcasting power stretch factors. We also found that although the sink structure $YG^*(V)$ has both bounded node degree and unicast power stretch factor theoretically, it is not dramatically better than $YY(V)$ in practice.

It is an open problem whether $YY(V)$ has a bounded unicasting power stretch factor theoretically. We also leave it as a future work to design an efficient localized algorithm achieving the following three objectives: a constant bounded node degree, a constant bounded unicasting power stretch factor, and a constant bounded broadcasting power stretch factor.

One of the main questions remaining to be studied is to integrate the overhead cost of transmission. In this paper, we assume that the power needed to support a link uv is $\|uv\|^\beta$ for a real constant $2 \le \beta \le 5$. However, this model does not fully reflect the actual transmission cost, which is often $\|uv\|^\beta + c$. Here c is a real positive constant. We leave it as a future work to design an efficient algorithm to construct a power-efficient topology by considering this cost overhead c.

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